

# Pion Electromagnetic Form Factor up to 10 [GeV/c]<sup>2</sup>

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## Abstract

The light-front approach is applied to calculate the electromagnetic current for quark-antiquark bound states for the pion. The pion electromagnetic form factor is obtained from the "+" and "-" component of the electromagnetic current in the Drell-Yan frame, with different models of the  $\pi - q\bar{q}$  vertex and the results for the pion electromagnetic form factor are compared with the experimental data up to 10 [GeV/c]<sup>2</sup> and another hadronic models. The rotational symmetry properties of the pion electromagnetic current related with the zero-modes in the light-front are investigated.

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## I. INTRODUCTION

The quantum chromodynamics,  $QCD$ , is believed the correct theory of the strong interactions and the non-perturbative regime of the  $QCD$  is the most important question not solved yet. However, with the relativistic constituents quark model,  $RCQM$ , is possible to give answers to hadronic physics, in terms the degrees of freedom from the  $QCD$ , ie., quarks and gluons [1]. The main proposal with the light-front here, are try describe in an consistent way the hadronic bound state systems to both high and low  $Q^2$  regime. For this purpose, the light-front quantization is utilized to compute the hadronic bound state wave function, which is simpler to make the calculations, compared to the instant form quantum field theory [1, 2]. In the light-front, the bound state wave functions are defined in the hypersurface  $x^+ = x^0 + x^3 = 0$ , and, theses wave functions, are covariant under kinematical front-form boosts, because of the stability of the Fock-state decomposition [3]. The bound state wave functions, with the light-front constituent quark model (LFCQM) in the light-front approach have received much attention lately [4, 5].

The LFCQM models have an impressive successful in the describe the electromagnetic properties of the hadronic wave functions, for pseudoscalar particles and vector particles [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. However, the extraction of the electromagnetic form-factor in the light-front approach depends which component of the electromagnetic current is utilized to calculate the form-factors, because the problems related with the rotational symmetry breaking [11, 23, 24]. It is was found in the references [11, 16, 23] for spin-1 particles, the plus ("J<sup>+</sup>") component of the electromagnetic current is not free of the pair terms contribution in the Breit frame ( $q^+ = 0$ ) and the rotational symmetry is broken. Then, the electromagnetic matrix elements of the current with the light-front approach have another contributions, besides of the valence contribution to the electromagnetic current. That contribution correspond the pair terms contribution for the matrix elements of the electromagnetic current [23, 24, 25]. If the pair terms contribution is taken correct, no matter which component of the electromagnetic current is utilized in the light-front approach in order to extract the electromagnetic form factors of the hadronic bound states. In this work, two types of the vertex function are utilized in order to calculated the pion electromagnetic form-factor for the  $\pi - q\bar{q}$  vertex and compared with the new experimental data [26, 27, 28, 29, 30]. At low momentum transfer, non-perturbative regime

of the QCD (quantum chromodynamics) is more important when compared with the higher momentum transfer for the perturbative regime of the QCD. Perturbative QCD work well after  $1.0 \text{ (GeV/c)}^2$  and dominate near  $5.0 \text{ (GeV/c)}^2$ . The light bound state mesons, like the pion and rho meson, are described with another approaches in the references [31, 32, 33, 34, 35, 36, 37, 38]. Another possibility is to study the light bound state meson, like pion and rho meson with the lattice formulate in the light-front [39]. For the lightest bound state meson, the models with the Schwinger-Dyson equations [31, 32, 33] describe the electromagnetic form factor quite very well, however some difference between the models in the literature exist [40]. In this paper, the light-front models for the pion present at previous work [12, 25] are extended to higher momentum transfer (up to  $10 \text{ (GeV/c)}^2$ ) and compared with another quarks models, ie., QCD sum rules [41, 42] and vector meson dominance [43, 44, 45]. This paper is presented with following sections: section II, the model of the wave function for the bound state quark-antiquark in the light-front are presented and the electromagnetic form factor are calculated with the vertex  $\pi - q\bar{q}$  and with another models. In the section III, the numerical results and discussions are given. Finally the conclusions are presented in the section IV.

## II. LIGHT-FRONT WAVE FUNCTION AND ELECTROMAGNETIC FORM FACTOR

In the light-front approach, the main goal to the bound state problem are solve the following equation, ie., the bound state equation:

$$H_{LF}|\Psi\rangle = M^2|\Psi\rangle. \quad (1)$$

In the equation above, the light-front Hamiltonian  $H_{LF}$ , have the eigenvalues given by the invariant mass  $M^2$ , where the eigenvalues are associate with the physical particles, the eigenstates of the light-front Hamiltonian [1]. The hadronic light-front wave functions are related with Bethe-Salpeter equations, ie., Bethe-Salpeter wave function (see the ref. [25] for details about this point). With the light-front wave function, is possible to calculated the matrix elements of the current between hadronic bound states. In the light-front, the meson bound state wave function are superpositions for all Fock states and the wave function are

given by

$$|\Psi_{meson} \rangle = \Psi_{q\bar{q}}|q\bar{q} \rangle + \Psi_{q\bar{q}g}|q\bar{q}g \rangle + \dots \quad (2)$$

With the light-front hadronic wave function, is possible to calculate the hadronic electromagnetic form factors, from the overlap of light-front wave function in the final and the initial state.

In general, the electromagnetic form-factor for the pion is expressed with the covariant equation below

$$(p + p')^\mu F_\pi(Q^2) = \langle \pi(p') | J^\mu | \pi(p) \rangle, \quad Q = p' - p, \quad (3)$$

where  $J^\mu$  is the electromagnetic current, which is possible to express in terms of the quarks fields  $q_f$  and charge  $e$  ( $f$  is the flavor of the quark field):  $J^\mu = \sum_f e_f \bar{q}_f \gamma_\mu q_f$ . The electromagnetic matrix elements of the current, are writing in the follow equation

$$J^\mu = -i2e \frac{m^2}{f_\pi^2} N_c \int \frac{d^4k}{(2\pi)^4} Tr [S(k) \gamma^5 S(k - p') \gamma^\mu S(k - p) \gamma^5] \Gamma(k, p') \Gamma(k, p), \quad (4)$$

where  $S(p) = \frac{1}{\not{p} - m + i\epsilon}$  is the quark propagator and  $N_c = 3$  is the colors numbers. The calculation here, is performade in the Breit frame with  $p^\mu = (0, q/2, 0, 0)$ ,  $p'^\mu = (0, q/2, 0, 0)$  for the initial and final momenta of the system respectively and the momentum transfer is  $q^\mu = (0, q, 0, 0)$  and  $k^\mu$  is the spectator quark momentum. The factor 2 appear from the isospin algebra. In this model, the electromagnetic current is conserved, which is easy to prove in the Breit frame.

The function  $\Gamma(k, p)$  are the regulator vertex function, used in order to regularize the Feynman triangle diagram, Eq. (4), for the electromagnetic current. Here, we have utilized two possible  $q\bar{q}$  vertex function; the first one, is the nonsymmetric vertex, utilized in a previous work [12]

$$\Gamma^{NSY}(k, p) = \left[ \frac{N}{((p - k)^2 - m_R^2 + i\epsilon)} \right]. \quad (5)$$

and the second, a symmetric vertex [25]

$$\Gamma^{SY}(k, p) = \left[ \frac{N}{(k^2 - m_R^2 + i\epsilon)} + \frac{N}{((p - k)^2 - m_R^2 + i\epsilon)} \right]. \quad (6)$$

The  $J^+$  component of the electromagnetic current is utilized in order to extracted the pion electromagnetic form factor from the Eq. (3), where the Dirac "plus" matrix is given by

$\gamma^+ = \gamma^0 + \gamma^3$ . The component  $J_\pi^+$  of the electromagnetic current for the pion meson is calculated with the triangle Feynman diagram, which represent the foton absorption process by the hadronic bound state of the  $q\bar{q}$  pair:

$$\begin{aligned} J_\pi^+ &= e(p^+ + p'^+)F_\pi(q^2) \\ &= ie\frac{m^2}{f_\pi^2}N_c \int \frac{dk^- dk^+ d^2k_\perp}{2(2\pi)^4} \frac{Tr^+[\ ]\Gamma(k, p')\Gamma(k, p)}{k^+(k^- - \frac{f_1 - i\epsilon}{k^+})} \\ &\quad \times \left[ \frac{1}{(p^+ - k^+)(p^- - k^- - \frac{f_2 - i\epsilon}{p^+ - k^+})(p'^+ - k^+)(p'^- - k^- - \frac{f_3 - i\epsilon}{p'^+ - k^+})} \right]. \end{aligned} \quad (7)$$

where,  $f_1 = k_\perp^2 + m^2$ ,  $f_2 = (p - k)_\perp^2 + m^2$  and  $f_3 = (p' - k)_\perp^2 + m^2$ .

The Dirac trace in the equation above, is writing in the light-front coordinates as (see the appendix for the light-front review):

$$\begin{aligned} Tr^+[\ ] &= [(\not{k} + m)\gamma^5(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\gamma^5] = \\ &= [-4k^-(k^+ - p^+)^2 + 4(k_\perp^2 + m^2)(k^+ - 2p^+) + k^+q^2]. \end{aligned}$$

The quadri-momentum integration of the Eq. (7), have two intervals contribution: (i)  $0 < k^+ < p^+$  and the second (ii)  $p^+ < k^+ < p'^+$ , where  $p'^+ = p^+ + \delta^+$ . The first interval is the contribution of the valence wave function for the electromagnetic form factor and the second interval correspond the pair terms contribution to the matrix elements of the electromagnetic current. In the case of the nonsymmetric vertex with the plus component of the electromagnetic current, the second interval not give any contribution for the matrix elements of the current.

But is not the case for the minus component of the electromagnetic current for the pion, where, besides the valence contribution, we have a non-valence contribution (see the reference [12], for details). In the first interval integration, the pole contribution is  $\bar{k}^- = \frac{f_1 - i\epsilon}{k^+}$ , then, the electromagnetic form factor obtained are:

$$\begin{aligned} F_\pi^{+(i)(NSY)}(q^2) &= 2ie\frac{m^2 N^2}{2p^+ f_\pi^2}N_c \int \frac{d^2k_\perp dk^+}{2(2\pi)^4} \left[ \frac{-4\bar{k}^-(k^+ - p^+)^2 + 4(k_\perp^2 + m^2)(k^+ - 2p^+) + k^+q^2}{k^+(p^+ - k^+)^2(p^+ - k^+)^2} \right. \\ &\quad \left. \frac{\theta(k^+)\theta(P^+ - k^+)}{(p^- - \bar{k}^- - \frac{f_2 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_3 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_4 - i\epsilon}{p^+ - k^+})(P'^- - \bar{k}^- - \frac{f_5 - i\epsilon}{p'^+ - k^+})} \right]. \end{aligned} \quad (8)$$

where, the functions  $f_1$ ,  $f_2$  and  $f_3$  are already defined above and the new functions are  $f_4 = (p - k)_\perp^2 + m_R^2$  and  $f_5 = (p' - k)_\perp^2 + m_R^2$ . After the integration in the light-front energy

$k^-$ , the equation for the electromagnetic form factors with nonsymmetric vertex (and the "plus component" of the electromagnetic current) is given by

$$F_\pi^{+(i)(NSY)}(q^2) = ie \frac{m^2 N^2}{p^+ f_\pi^2} N_c \int \frac{d^2 k_\perp dk^+}{2(2\pi)^4} \left[ \frac{-4\bar{k}^-(k^+ - p^+)^2 + 4(k_\perp^2 + m^2)(k^+ - 2p^+) + k^+ q^2}{k^+(p^+ - k^+)^2(p^+ - k^+)^2(p^- - \bar{k}^- - \frac{f_2 - i\epsilon}{p^+ - k^+})} \right. \\ \left. \frac{\theta(k^+)\theta(p^+ - k^+)}{(p^- - \bar{k}^- - \frac{f_3 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_4 - i\epsilon}{p^+ - k^+})(p'^- - \bar{k}^- - \frac{f_5 - i\epsilon}{p'^+ - k^+})} \right]. \quad (9)$$

The light-front wave function for the pion with the nonsymmetric vertex is writing below as

$$\Psi^{(NSY)}(x, k_\perp) = \left[ \frac{N}{(1-x)^2(m_\pi^2 - \mathcal{M}_0^2)(m_\pi^2 - \mathcal{M}_R^2)} \right], \quad (10)$$

where the fraction of the momentum carried by the quark is  $x = k^+/p^+$  and the  $\mathcal{M}_R$  function is writing below as

$$\mathcal{M}_R^2 = \mathcal{M}^2(m^2, m_R^2) = \frac{k_\perp^2 + m^2}{x} + \frac{(p - k)_\perp^2 + m_R^2}{(1-x)} - p_\perp^2. \quad (11)$$

In the pion wave function, the free mass operator is  $\mathcal{M}_0^2 = \mathcal{M}^2(m^2, m^2)$  and the normalization constant  $N$  is found with the condition  $F_\pi(0) = 1$ .

The final pion electromagnetic form factor expressed with the light-front wave function above is:

$$F_\pi^{+(i)(NSY)}(q^2) = \frac{m^2}{p^+ f_\pi^2} N_c \int \frac{d^2 k_\perp dx}{2(2\pi)^3 x} \left[ -4\left(\frac{f_1}{xp^+}\right)(xp^+ - p^+)^2 + 4f_1(xp^+ - 2p^+) \right. \\ \left. + xp^+ q^2 \right] \Psi_f^{*(NSY)}(x, k_\perp) \Psi_i^{(NSY)}(x, k_\perp) \theta(x) \theta(1-x). \quad (12)$$

In the light-front approach, besides the valence contribution to the electromagnetic current, the non-valence components give contribution to the electromagnetic current [11, 12, 24]. The non-valence components or the pair term contribution, is calculated in the second interval of the integration (ii) with the "dislocation pole method", developed in the references [12, 23, 24], for the "plus component" of the electromagnetic current, the non-valence contribution to the electromagnetic form factor are given by

$$F_\pi^{+(ii)(NSY)}(q^2) = \lim_{\delta^+ \rightarrow 0} 2ie \frac{m^2 N^2}{2p^+ f_\pi^2} N_c \int \frac{d^2 k_\perp dk^+}{2(2\pi)^4} \left[ \frac{-4\bar{k}^-(k^+ - p^+)^2 + 4(k_\perp^2 + m^2)(k^+ - 2p^+) + k^+ q^2}{k^+(p^+ - k^+)^2(p^+ - k^+)^2} \right. \\ \left. \frac{\theta(p^+ - k^+)\theta(p'^+ - k^+)}{(p^- - \bar{k}^- - \frac{f_2 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_3 - i\epsilon}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_4 - i\epsilon}{p^+ - k^+})(p'^- - \bar{k}^- - \frac{f_5 - i\epsilon}{p'^+ - k^+})} \right] \\ \propto \delta^+ = 0. \quad (13)$$

In the equation above, the electromagnetic form factor is directly proportional to the term  $\delta^+$  and with that term go to zero, then, the non-valence or the pair term contribution for the pion electromagnetic form factor is zero with the nonsymmetric vertex [12].

In the following, with the minus component of the electromagnetic current ( $J_\pi^-$ ), we extract the electromagnetic pion form factor with nonsymmetric vertex, Eq. (5). In this case, we have two contribution, one is the valence contribution for the wave function and the second is the pair terms contribution or nonvalence contribution to the electromagnetic matrix elements of the current [12, 24, 25]. The pion electromagnetic form factor for the minus component of the electromagnetic current, (here the  $J_\pi^-$  is related with the Dirac matrix by  $\gamma^- = \gamma^0 - \gamma^3$ , see the appendix) and the nonsymmetric vertex is given in the following equation

$$\begin{aligned} J_\pi^{-(NSY)} &= e(p+p')^- F_\pi^{-(NSY)}(q^2) \\ &= ie^2 \frac{m^2}{f_\pi^2} N_c \int \frac{d^4 k}{(2\pi)^4} Tr \left[ \frac{\not{k} + m}{k^2 - m^2 + i\epsilon} \gamma^5 \frac{\not{k} - \not{p}' + m}{(p' - k)^2 - m^2 + i\epsilon} \gamma^- \frac{\not{k} - \not{p} + m}{(p - k)^2 - m^2 + i\epsilon} \gamma^5 \right] \\ &\quad \times \left[ \Gamma(k, p') \Gamma(k, p) \right]. \end{aligned} \quad (14)$$

The Dirac trace in the equation Eq. (14), calculated with the light-front formalism, result in the following expression:

$$\begin{aligned} Tr^-[\ ] &= [-4k^- k^+ - 4p^+(2k_\perp^2 + k^+ p^+ + 2m^2) \\ &\quad + k^-(4k_\perp^2 + 8k^+ p^+ + q^+ + 4m^2)]. \end{aligned} \quad (15)$$

In order to calculated the pair terms contribution for the minus component of the electromagnetic current in the second interval integration ( $p^+ < k^+ < p'^+$ ), the  $k^-$  dependence in the trace is performade and the pair terms matrix elements are build as:

$$\begin{aligned} J^{-(ii)}(NSY) &= \lim_{\delta^+ \rightarrow 0} 2ie \frac{m^2}{f_\pi^2} N_c \int \frac{d^2 k_\perp dk^+}{2(2\pi)^4} \left[ \frac{-4\bar{k}^- k^+ + \bar{k}^- 4(k_\perp^2 + 8k^+ p^+ + q^+ + 4m^2)}{k^+(p^+ - k^+)(p'^+ - k^+)(\bar{k}^- - \frac{f_{1-i\epsilon}}{k^+})} \right. \\ &\quad \left. \frac{\theta(p^+ - k^+) \theta(p'^+ - k^+)}{(p^- - \bar{k}^- - \frac{f_{2-i\epsilon}}{p^+ - k^+})(p^- - \bar{k}^- - \frac{f_{4-i\epsilon}}{p^+ - k^+})(p'^- - \bar{k}^- - \frac{f_{5-i\epsilon}}{p'^+ - k^+})} \right], \end{aligned} \quad (16)$$

where  $p'^+ = p^+ + \delta^+$  and  $\bar{k}^- = p^- - \frac{f_{3-i\epsilon}}{p'^+ - k^+}$ . The pair terms contribution for the minus component of the electromagnetic current is obtained with the equation above and the Breit frame is recovered in the limit  $\delta^+ \rightarrow 0$ :

$$J_{\pi}^{-(ii) (NSY)} = 4\pi \left( \frac{m_{\pi}^2 + q^2/4}{p^+} \right) \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \sum_{i=2}^5 \frac{\ln(f_i)}{\prod_{j=2, i \neq j}^5 (-f_i + f_j)}. \quad (17)$$

The pair terms contribution to the pion electromagnetic form factor is build with the minus component of the matrix elements for the electromagnetic current calculated above:

$$F_{\pi}^{-(ii) (NSY)}(q^2) = \frac{N^2}{2p^-} \frac{m^2}{f_{\pi}^2} N_c \left( 4\pi \frac{m_{\pi}^2 + q^2/4}{p^+} \right) \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \sum_{i=2}^5 \frac{\ln(f_i)}{\prod_{j=2, i \neq j}^5 (-f_i + f_j)}. \quad (18)$$

The full form factor are the sum of the partial form factors  $F_{\pi}^{-(i)}$  and  $F_{\pi}^{-(ii)}$ :

$$F_{\pi}^{-(NSY)}(q^2) = F_{\pi}^{-(i)(NSY)}(q^2) + F_{\pi}^{-(ii)(NSY)}(q^2). \quad (19)$$

If the pair terms is not take in account, the rotational symmetry is broken and the covariance was lost for the  $J_{\pi}^{-}$  component of the electromagnetic current, (see also the Fig. 1 ). With the pair terms contribution, the following identity are obtained

$$F_{\pi}^{-(NSY)}(q^2) = F_{\pi}^{+(NSY)}(q^2). \quad (20)$$

and the full covariance is restorate.

In the next step, the model utilized is the symmetric vertex  $\pi$ - $q\bar{q}$ , with the plus component ”+” of the electromagnetic current, Eq. (6), utilized in the reference [25].

This vertex is symmetric by the exchange of the quadri-momentum for the quark and the antiquark and in the light-front coordinates is writing in the following way:

$$\begin{aligned} \Gamma(k, p) &= \mathcal{N} \left[ k^+ \left( k^- - \frac{k_{\perp}^2 + m_R^2 - i\epsilon}{k^+} \right) \right]^{-1} + \\ &\mathcal{N} \left[ (p^+ - k^+) \left( p^- - k^- - \frac{(p - k)_{\perp}^2 + m_R^2 - i\epsilon}{p^+ - k^+} \right) \right]^{-1}. \end{aligned} \quad (21)$$

With the symmetric vertex, Eq. (21), the pion valence wave function result in the following expression

$$\Psi^{(SY)}(k^+, \vec{k}_{\perp}) = \left[ \frac{\mathcal{N}}{(1-x)(m_{\pi}^2 - \mathcal{M}^2(m^2, m_R^2))} + \frac{\mathcal{N}}{x(m_{\pi}^2 - \mathcal{M}^2(m_R^2, m^2))} \right] \frac{p^+}{m_{\pi}^2 - M_0^2}. \quad (22)$$

The electromagnetic form factor for the pion valence wave function, Eq. (22), calculated in the Breit frame ( $q^+ = 0$ ) are



$$F_\pi^{(SY)}(q^2) = \frac{m^2 N_c}{p^+ f_\pi^2} \int \frac{d^2 k_\perp}{2(2\pi)^3} \int_0^1 dx \left[ k_{on}^- p^{+2} + \frac{1}{4} x p^+ q^2 \right] \frac{\Psi_f^{*(SY)}(x, k_\perp) \Psi_i^{(SY)}(x, k_\perp)}{x(1-x)^2}. \quad (23)$$

The normalization constant  $\mathcal{N}$  is determined from the condition  $F_\pi^{SY}(0) = 1$ . The pion electromagnetic form factor, calculated with the symmetric wave function are presented in the Fig. 1 for higher momentum transfer (up to 10  $(GeV/c)^2$ ) and in the figure Fig. 2 to lower momentum (up to 0.5  $(GeV/c)^2$ ). In both regimes, the differences between the symmetric and non-symmetric vertex are not to bigger.

The next models discussed are de *QCD Sum Rules* and the *vector meson dominance model (VMD)*. *QCD Sum Rules* model are presented, in order to calculated the pion electromagnetic form factor and compared with the light-front models discussed in the last sections.

With the *QCD Sum Rules* model, *QCDSR*, the pion electromagnetic form factor are obtained directly and is not necessary to determine the wave function to the hadron considered [41, 42]. In the following, the electromagnetic form factor for the pion, is calculated with the *QCDSR* [42]:

$$F_\pi^{LD,soft}(q^2) = 1 - \left( \frac{1 - 6s_0/q^2}{1 + 4s_0/q^2} \right), \quad (24)$$

and

$$F_\pi^{LD,\alpha_s}(q^2) = \left( \frac{\alpha_s}{\pi} \right) \frac{1}{1 + q^2/2s_0}. \quad (25)$$

where the values utilized here are  $s_0 = 4\pi^2 f_\pi = 0.67 \text{ GeV}^2$ ,  $(\alpha_s/\pi) = 0.1$  and the  $f_\pi$  utilized is the experimental value 0.093  $\text{GeV}$ . The full electromagnetic pion form factor are the sum for two contributions given above [42]:

$$F_\pi^{QCDSR}(q^2) = F_\pi^{LD,soft}(q^2) + F_\pi^{LD,\alpha_s}(q^2). \quad (26)$$

Besides the light-front models and the *QCDSR*, the vector meson dominance hypothesis, (*VMD*) [18, 43, 44, 45], is show in this work.

$$F_\pi^{VMD}(q^2) = \frac{1}{1 + \frac{q^2}{m_\rho^2}}, \quad (27)$$

where, in the Eq. (27), the the rho meson mass utilized is close to the experimental value,  $m_\rho = 0.77 \text{ GeV}$  and the results are present at the Fig. 1.

In the case presented here, only the lightest vector resonance ( $m_\rho$ ) is take account in the monopole model of the *VMD*, Eq. (27). The vector meson dominance work quite well in the timelike regime below the  $\pi\pi$  threshold. At low energies, ie., spacelike regime, the vector meson dominance model give a reasonable description for the pion electromagnetic form factor. See the Fig. 1 and reference [43, 45] for details about the VMD .

### III. RESULTS

In the case of the nonsymmetric vertex, the pion radius is utilized to fix the parameters of the model. The parameters are the quark mass  $m_q = 0.220$  GeV and the regulator mass  $m_R = 0.946$  GeV. The pion mass utilized is the experimental value,  $m_\pi = 0.140$  GeV. The experimental radius of the pion is  $r_{exp} = 0.67 \pm 0.02$  fm [26]. The calculation of the pion decay constant with this model of the vertex (nonsymmetric) and parameters above, produce the pion decay constant like  $f_\pi = 101$  MeV, close with the experimental value ( $\simeq f_\pi = 93.0$  MeV).

In the case of the symmetric vertex, the parameters are the quark mass  $m_q = 0.220$  GeV, regulator  $m_R = 0.60$  GeV and the experimental mass of the pion, 0.140 GeV. Our choice of the regulator mass value fit the pion decay constant value,  $f_\pi^{exp} = 92.4$  MeV for the symmetric vertex.

Both models in the light-front with symmetric and nonsymmetric vertex have agreement with the experimental data at low energy, however some differences are found up to  $2.0 (GeV/c)^2$ . The experimental data from reference [28] are describe up  $10 (GeV/c)^2$  with the symmetric vertex. For the minus component of the electromagnetic current, the pair terms or non-valence terms is essential to obtain the full covariant pion electromagnetic form factor.

The models of the  $q\bar{q}$  vertex (symmetric and nonsymmetric ) in the light-front are compared with the vector meson dominance (VMD) and QCD sum rules [41] in the figures 1 and 2 for higher and low momentum transfer.

At very low momentum transfer, QCD sum rules not have good agreement with experimental data [27]. In that region, light-front models presented here, give better agreement with experimental data [26, 27, 28, 29].

The ratios between the electromagnetic current in the light-front and the electromagnetic

current calculated in the instant form, are given in the following equations

$$\begin{aligned}
Ra^I &= \frac{J_{LF}^+}{J_{Cov}^+}, & Ra^{II} &= \frac{J_{LF}^-}{J_{Cov}^-}, \\
Ra^{III} &= \frac{J_{LF}^- + J_{LF}^{-(Pair)}}{J_{Cov}^-}, & Ra^{IV} &= \frac{J_{LF}^-}{J_{Cov}^+}, \\
Ra^V &= \frac{J_{LF}^- + J_{LF}^{-(Pair)}}{J_{Cov}^+}, & &
\end{aligned} \tag{28}$$

where the nonsymmetric vertex is utilized, Eq. (5). The first equation above,  $(Ra^I)$ , is the plus component of the electromagnetic current calculated in the light-front approach divided by instant form formalism. Because the pair terms not give contribution for the plus component of the electromagnetic current, the ratios  $(Ra^I)$  are constant (see also Fig. 2).

The second ratios,  $(Ra^{II})$ , are the minus component of the electromagnetic current,  $J^-$ , calculated with the light-front approach and divided by the electromagnetic current calculated in the instant form formalism. In  $Ra^{III}$  ratios, the pair terms contribution to the electromagnetic is included, then, the covariance are restorate. The ratios  $Ra^{IV}$  and  $Ra^V$  are the "minus" component of the electromagnetic current without and with the pair terms contribution divided by the "plus" component of the electromagnetic current calculated in the instant form approach.

Is showed at the Fig. 3, in a clear way, the explanation for the broken of the rotational symmetry in the light-front approach, it is because the pair terms or non-valence contribution for the electromagnetic current. The restoration of the symmetry breaking are presented by add the pair terms contribution to the minus component of the electromagnetic current calculated in the light-front.

#### IV. CONCLUSION

The light-front approach is a natural way to describe relativistic systems, ie., relativistic bound states, like the pion. With the light-front, is possible to calculated the electromagnetic form-factors for bound state and compare the experimental data. However, problems related with the broken of the rotational symmetry in the light-front approach are important and the pair terms or no-valence terms contribution for the covariance restoration in higher energies is also necessary. After the pair terms inclusion in the matrix elements of the electromagnetic current, the covariance are complete restorate and no matter with the component of the

electromagnetic current are utilized in order to extract the pion form factor with the light-front approach (see Fig. 1 and Fig. 3).

The comparison with the light-front models for the vertex  $\pi - q\bar{q}$  and another hadronic models are presented in the Fig. 1 and the pion electromagnetic form-factor is very well described for hadronic models presented in this work, however some differences exist between the models after  $2 (GeV/c)^2$  in the higher energy regime.

At low energy regime, ie., up to  $0.5 (GeV/c)^2$ , the differences between QCD sum rules and another hadronic models is more evident and important (see the Fig. 2). In the low energy, the QCDSR, not describe the experimental data very well. Since the pion electromagnetic form-factor are sensitive to the quark model utilized, is important to compare different models with new experimental data.

In conclusion, with the light-front approach and the vertex models for  $\pi - q\bar{q}$  utilized in the present work, is possible describe the new experimental data for the pion electromagnetic form factor quiet very well up to  $10 (GeV/c)^2$ .

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### APPENDIX: LIGHT-FRONT

Some aspects of the light-front formalism utilized are presented in this appendix, more details about light-front quantum field theory are found in the ref's. [1, 2] .

In the light-front, one describe a system by its evolution in the time  $x^+$ . The usual instant form coordinates are related with the light-front coordinates by:

$$x^+ = x^0 + x^3, \quad x^- = x^0 - x^3, \quad \vec{x}_\perp = (x_1, x_2) . \quad (\text{A.1})$$

With that definitions for the light-front coordinates, the scalar product in the light-front approach are given by:

$$x \cdot y = \frac{1}{2}(x^+y^- + x^-y^+) - \vec{x}_\perp \cdot \vec{y}_\perp . \quad (\text{A.2})$$

The momentum coordinates in the light-front approach are:

$$\begin{aligned}k^+ &= k^0 + k^3 \\k^- &= k^0 - k^3 \\k_\perp &= (k_1, k_2) .\end{aligned}\tag{A.3}$$

With the light-front coordinates above, the integral phase space are given by

$$d^4k = \frac{1}{2}d^2k_\perp dk^+ dk^- \tag{A.4}$$

Dispersion energy-momentum for a free particle with mass  $m$  is

$$k^- = \frac{k_\perp^2 + m^2}{k^+} . \tag{A.5}$$

That dispersion relation is drastically different when compared with the instant form dispersion relation, where dispersion relation energy is not linear.

The Dirac matrix in the light-front quantum field theory are:

$$\begin{aligned}\gamma^+ &= \gamma^0 + \gamma^3 \\ \gamma^- &= \gamma^0 - \gamma^3 \\ \gamma_\perp &= (\gamma_1, \gamma_2) .\end{aligned}\tag{A.6}$$

The matrix elements of the electromagnetic current components  $(J^+, J^-, J_\perp)$  are related directly with Dirac gamma matrix expressed in the light-front matrix basis.

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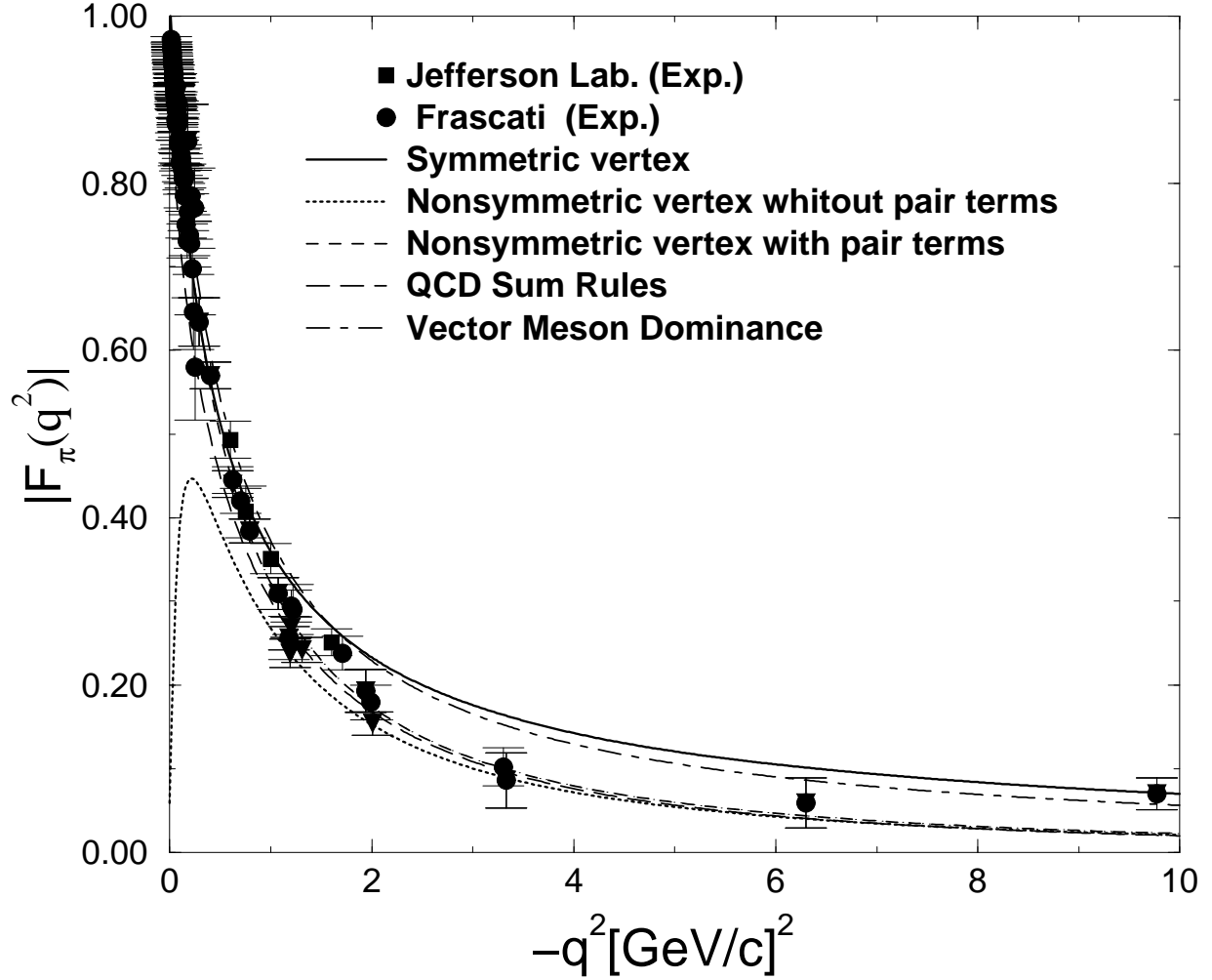


FIG. 1: Pion electromagnetic form factor compared with experimental data from new experimental data [29] (square) and [28] (circle). Solid line, full covariant form factor with  $J_\pi^+$  (symmetric vertex for the  $\pi - q\bar{q}$ ). The Dashed line are the form factor with  $J_\pi^-$  plus pair terms contribution and dotted line is the pion form factor without the pair terms contribution with the minus component of the electromagnetic current, both curves with the nonsymmetric vertex.



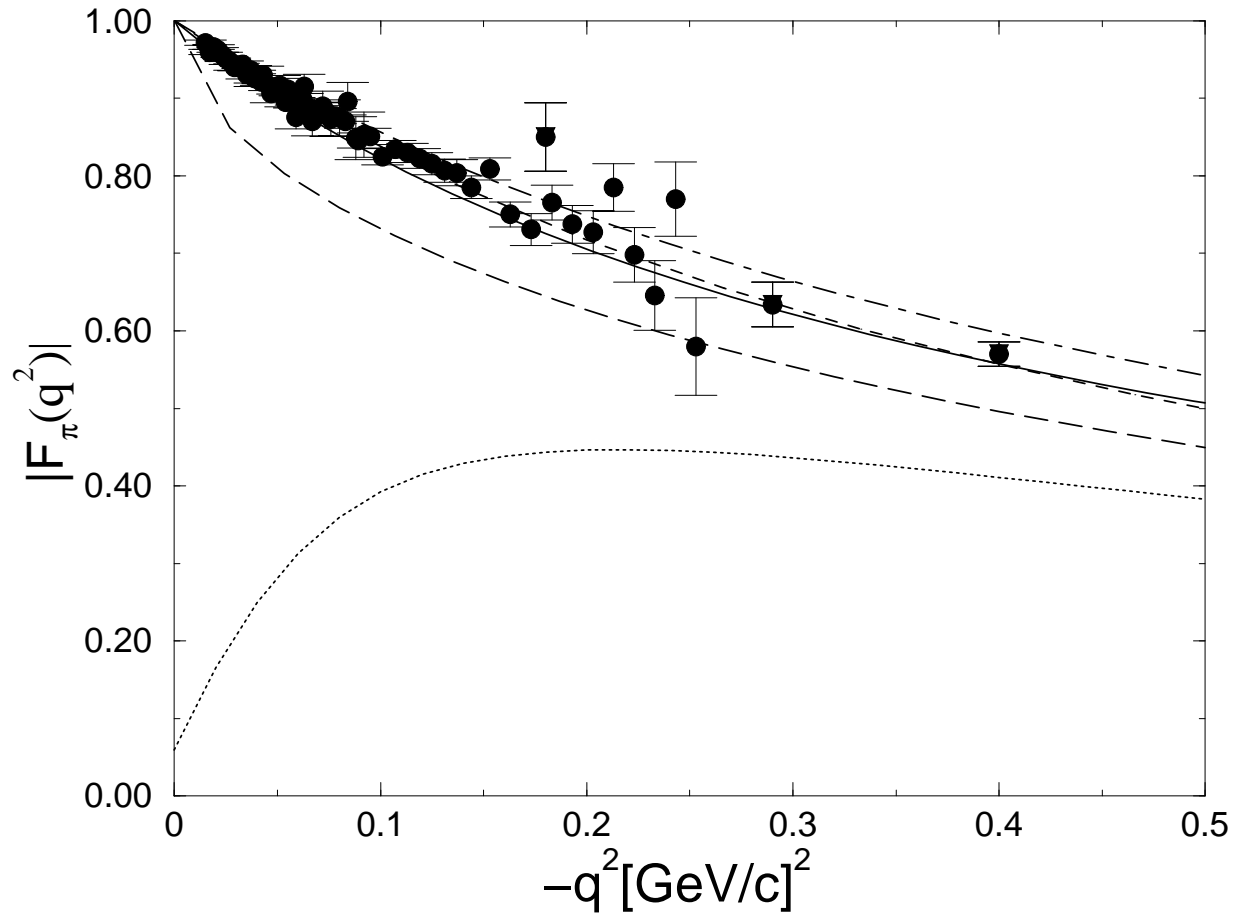


FIG. 2: Pion electromagnetic form factor squared for small  $Q^2$ ; labels are the same in the Fig. 1. Experimental data are in the Ref. [27, 28, 29].

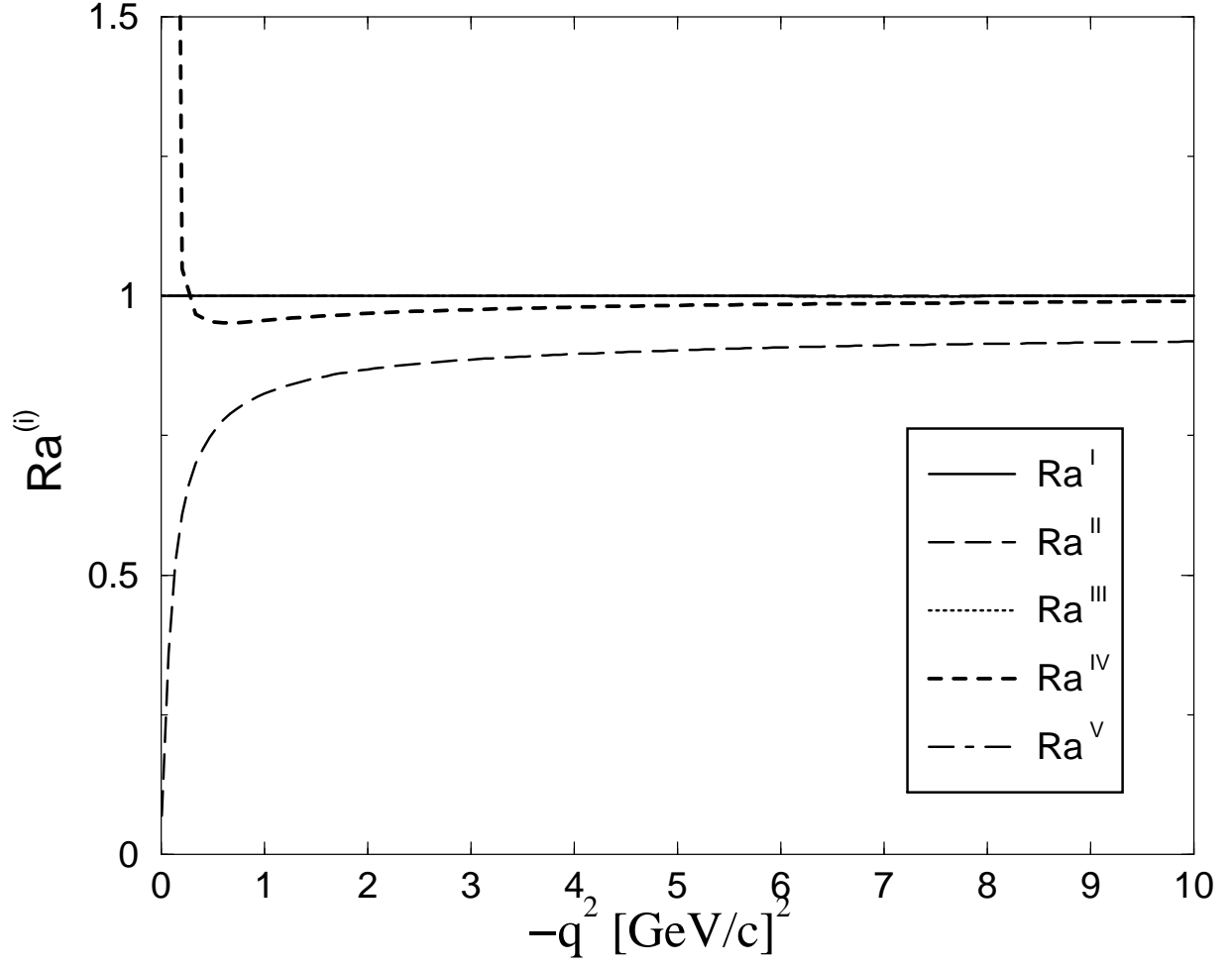


FIG. 3: Pion electromagnetic current ratios, Eq. 28.